Below is the mathematical model for the traditional production system without returns options (TPS) of domestic electric storage waters heaters (DESWH), which is introduced in:

Devoto C, Fernández E, 2019, "*Recovery of used products and an application in the Uruguayan industry*" (in spanish), Bachelor thesis, Faculty of Engineering, Universidad de la República, Uruguay, https://hdl.handle.net/20.500.12008/20592.

The model considers the costs of production and for holding inventory as well as the costs related to the procurement of raw materials to different suppliers and for safety stocks. Returns of used products are not considered.

Sets:

- TC: Set of DESWH with copper storage tank.
- TA: Set of DESWH with steel storage tank.
- $T = TC \cup TA$: Set of all DESWH.
- TP: Set of DESWH that can be produced in the same period.
- MP: Set of raw materials.
- $MPD \subset MP$: Set of raw material with lot constraints.
- PR: Set of suppliers.
- PRM: Set of pairs (p, m) with $p \in PR$ and $m \in MP$ if material m is provided by supplier p.
- I: Set of periods.

Parameters:

- CF_t: Unit cost of production for product $t \in T$.
- K_t: Set-up cost of production for product $t \in T$.
- HP: Unit cost for holding inventory of final products. The same for all products.
- HM_m: Unit cost for holding inventory of raw material $m \in MP$.
- KM_p: Set-up cost of order for supplier $p \in PR$.
- D_{ti}: Demand of product $t \in T$ in period $i \in I$.
- R_{tm}: Amount of raw material $m \in MP$ required for the production of product $t \in T$.
- TN_t: Production time of product $t \in T$.
- TD: Available production time in the planning horizon.
- QP: Storage capacity for final products.
- QA: Storage capacity for raw materials.
- ST_t: Safety stock established for final product $t \in T$.
- SM_m: Safety stock established for raw material $m \in MP$.
- YM_m: Minimum quantity to be supplied for raw material $m \in MP$.
- Io_t: Initial inventory level of product $t \in T$.
- Imo_m: Initial inventory level of raw material $m \in MP$.
- E_m: Lot size for each raw material $m \in MPD$.
- A: A big number, with $A = \sum_{t \in T} \sum_{i \in I} D_{ti}$.
- N: A big number.

Variables:

- x_{ti}: Production quantity of product $t \in T$ in period $i \in I$.
- y_{mi}: Quantity of raw material $m \in MP$ to be ordered in period $i \in I$.
- s_{ti}: Inventory level of product $t \in T$ in period $i \in I$.
- z_{mi}: Inventory level of raw material $m \in MP$ in period $i \in I$.
- n_{mi}: Numbers of lots of raw material $m \in MP$ ordered in period $i \in I$.
- δ_{ti}: 1 if product $t \in T$ is produced in period $i \in I$, 0 otherwise.
- β_{mi}: 1 if raw material $m \in MP$ is ordered in period $i \in I$, 0 otherwise.
- γ_{pi}: 1 if raw material is ordered to supplier $p \in PR$ in period $i \in I$, 0 otherwise.

Mixed-Integer Linear Programming formulation for the TPS:

Min $\sum_{i\in I}\{\sum_{t\in T}(CF_t\, x_{ti} + K_t\delta_{ti} + HPs_{ti}) + \sum_{m\in MP} HM_m z_{mi} + \sum_{p\in PR} KM_p\gamma_{pi}\}$ (1)

subject to:

$x_{ti} \le A\delta_{ti,} \quad \forall t \in T,\ \forall i \in I$ (2)

$\delta_{ti,} \le x_{ti,} \quad \forall t \in T,\ \forall i \in I$ (3)

$\sum_{t\in T} x_{ti}\, TN_t \le TD, \quad \forall i \in I$ (4)

$y_{mi} \ge YM_m\beta_{mi}, \quad \forall m \in MP,\ \forall i \in I$ (5)

$y_{mi} \le N\beta_{mi}, \quad \forall m \in MP,\ \forall i \in I$ (6)

$s_{ti} = s_{(t,i-1)} - D_{ti} + x_{ti}, \quad \forall t \in T,\ \forall i \in I$ (7)

$s_{ti} \ge ST_t, \quad \forall t \in T,\ \forall i \in I$ (8)

$s_{t,0} = Io_t, \quad \forall t \in T$ (9)

$z_{mi} = z_{(m,i-1)} - \sum_{t\in T} R_{tm}x_{ti} + y_{mi}, \quad \forall m \in MP, \forall t \in T,\ \forall i \in I$ (10)

$z_{mi} \ge SM_m, \quad \forall m \in MP,\ \forall i \in I$ (11)

$z_{m,0} = Imo_m, \quad \forall m \in MP$ (12)

$\sum_{tc\in TC} \delta_{tc,i} \le 1, \quad \forall i \in I$ (13)

$\sum_{ta\in TA} \delta_{ta,i} \le 1, \quad \forall i \in I$ (14)

$\sum_{tx\in TA\setminus\{ta\}} \sum_{tp\in TP[tx]} \delta_{tp,i} \le 1 - \delta_{ta,i}, \quad \forall ta \in TA, \forall i \in I$ (15)

$y_{mi} = E_m n_{mi}, \quad \forall m \in MPD,\ \forall i \in I$ (16)

$$\gamma_{pi} \geq \beta_{mi}, \quad \forall p \in PR,\ \forall m \in PRM[p],\ \forall i \in I \tag{17}$$

$$\gamma_{pi} \leq \sum_{m \in MP} \beta_{mi}, \quad \forall p \in PR,\ \forall m \in PRM[p],\ \forall i \in I \tag{18}$$

$$\sum_{t \in T} s_{ti} \leq QP, \quad \forall i \in I \tag{19}$$

$$\sum_{m \in MP} \left(\frac{z_{mi}}{MA_m}\right) \leq QA,\ \forall i \in I \tag{20}$$

$$x_{ti}, s_{ti} \geq 0, \delta_{ti} \in \{0,1\}, \quad \forall t \in T,\ \forall i \in I \tag{21}$$

$$y_{mi}, z_{mi} \geq 0, \beta_{mi} \in \{0,1\}, \quad \forall m \in MP,\ \forall i \in I \tag{22}$$

$$n_{mi} \in \aleph^{+} \cup \{0\}, \quad \forall m \in MPD,\ \forall i \in I \tag{23}$$

The objective function of (1) is for minimizing the sum of the production of final products, the procurement of raw materials, and for holding inventories of both products and raw materials. Constraints (2) and (3) are for establishing that a production set-up cost is incurred if a positive amount of any product is produced in certain period. Constraints (4) state the production capacity in time terms. Constraints (5) and (6) establish that an order set-up cost is incurred if any positive amount of raw materials is ordered to certain supplier. Constraints (7) and (10) are the well-known inventory balance equations for final products and raw materials. Constraints (8) and (11) assure the safety stock levels and constraints (9) and (12) is for establishing the initial inventory level, for final products and raw material respectively. Constrains (13) to (15) state the products that can be produced in the same period. Constraints (16) assure the lot size restriction for raw materials. Constraints (17) and (18) are for representing if there is an order to certain supplier. Constraints (19) and (20) establish the limited storage for final products and raw materials. Constraints (21) to (23) state the set of values for the decision variables.